Geometric Sequences

SPI 3102.1.1: You will be able to interpret patterns found in sequences using variables.

A ***sequence*** is a set of numbers in a specific order. Each number in the sequence is a ***term***.

What is a geometric sequence??

- A sequence where each term is found by multiply the previous term by a constant. The constant being multiplied is called the ***common ratio (r)***. (Can't be found by using a 0 and r$\neq 0$ *or* 1)

Common Ratio: $r = \frac{term}{previous\ term}$

Example: 2, 6, 10, 14, 18, …

+4 +4 +4 +4

Since the difference between each number is +4, this is an arithmetic sequence and the common difference in 4.

Example 1: Determine whether each sequence is geometric. Explain.

a. 1, 5, 25, 125, 625, ...

×5 ×5 ×5 ×5 ×5 — Geometric; Common ratio: 5 (could also be found by $\frac{(term)5}{(previous\ term)1} = 5$)

b. 0, 5, 10, 15, 20, ...

+5 +5 +5 +5 — Not geometric; added 5 each time.

c. 1, -1, 1, -1, 1, …

×-1 ×-1 ×-1 ×-1 — Geometric; Common ratio: -1 (could also be found by $\frac{-1}{1} = -1$)

d. 1000, 200, 40, 8, ...

$\times\frac{1}{5}$ $\times\frac{1}{5}$ $\times\frac{1}{5}$ — Geometric; Common Difference: $\frac{1}{5}$ (could also be found by $\frac{200}{1000} = \frac{1}{5}$)

e. 56, -28, 14, -7, …

$\times-\frac{1}{2}$ $\times-\frac{1}{2}$ $\times-\frac{1}{2}$ — Geometric; Common Difference: $-\frac{1}{2}$ (could also be found by $\frac{-28}{56} = -\frac{1}{2}$)

Example 2: Find the next three terms.

a. 4, -8, 16, …. To get from number to number multiply by -2 each time.

Your sequence would be: 4, -8, 16, **-32, 64, -128**

b. 60, 72, 86.4, … To get from number to number multiply $\frac{6}{5}$ each time.

Your sequence would be: 60, 72, 86.4, **103.68, 124.416, 149.2992**

c. 64, 48, 36,… To get from number to number you multiply $\frac{3}{4}$.

Your sequence would be: 64, 48, 36, **27, 20.25, 15.1875**

Practice Problems!!! Book page 581 #16, 17, 19 (determine if the sequence is geometric or not geometric) and book page 581 #20, 23

Formula for the nth Term of a Geometric Sequence

$$a_n = a_1(r)^{n-1}$$

a_n: n^{th} term (what you are looking for)
a_1: first term
r: common ratio
n: what term you are looking for

Example 3:

A) Write an equation for the nth term of the geometric sequence: 3, -12, 48, -192, …

$a_1 = 3$ (because it's the first term)
$r = -4$ (because multiply -4 each time)

$a_n = a_1(r)^{n-1}$
$a_n = 3(-4)^{n-1}$ Plug 3 in for a_1 and -4 for r

B) Find the 7^{th} term

$a_n = 3(-4)^{n-1}$
$a_7 = 3(-4)^{7-1}$ Remember n is what you are looking for; so plug in 7 for n
$a_7 = 3(-4)^6$ Simplify: Subtract exponent
$a_7 = 3(4096)$ Simplify: Find $(-4)^6$. Make sure to put -4 in parentheses to get 4096.
$a_7 = 12288$ Simplify: Multiply

OR

Since the common ratio is -4, you can multiply -4 to the sequence until you reach the 7^{th} term.

3, -12, 48, -192, _____, ______, ______

a_1, a_2, a_3, a_4, a_5, a_6, a_7
×-4 ×-4 ×-4 ×-4 ×-4 ×-4

3, -12, 48, -192, 768, -3072, **12288**

So, $a_7 = 12288$

Example 4:

A) Write an equation for the nth term of the arithmetic sequence: 7, 21, 63, …

$a_1 = 7$	(because it's the first term)
$r = 3$	(because multiply 3 each time)

$a_n = a_1(r)^{n-1}$	
$a_n = 7(3)^{n-1}$	Plug 7 in for a_1 and 3 for r

C) Find the 9th term

$a_n = 7(3)^{n-1}$	
$a_7 = 7(3)^{8-1}$	Remember n is what you are looking for; so plug in 8 for n
$a_7 = 7(3)^7$	Simplify: Subtract exponent
$a_7 = 7(2187)$	Simplify: Find $(3)^7$.
$a_7 = 15309$	Simplify: Multiply

OR

Since the common ratio is 3, you can multiply 3 to the sequence until you reach the 8th term.

7, 21, 63, ______ , _______ , _______ , ________ , ________

$a_1, \ a_2, \ a_3, \ a_4, \ a_5, \ a_6 \ \ a_7, \ \ a_8$

×3 ×3 ×3 ×3 ×3 ×3 ×3

7, 21, 63, 189, 567, 1701, 5103, **15309**

So, $a_8 = 15309$

Practice Problems!! Book page 581 #28, 29 (write the equation 1st, then find the term.)
Complete this on the same sheet as your previous problems.