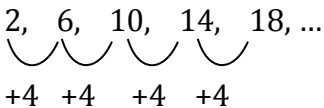# Arithmetic Sequences

SPI 3102.1.1: You will be able to interpret patterns found in sequences using variables.

A _**sequence**_ is a set of numbers in a specific order. Each number in the sequence is a _**term**_.

What is an arithmetic sequence??
- A sequence where the difference between the terms is constant. The difference between terms is called the _**common difference (d)**_.
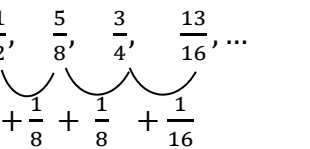
Example: 2, 6, 10, 14, 18, …
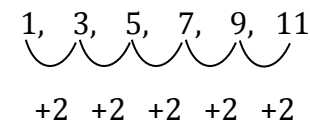
          +4 +4   +4   +4

Since the difference between each number is +4, this is an arithmetic sequence and the common difference in 4.
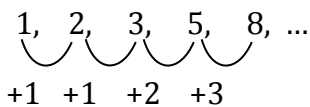
Example 1: Determine whether each sequence is arithmetic. Explain.

a. 0, 5, 10, 15, 20, …

    +5 +5 +5   +5   Arithmetic; Common Difference: 5

b. 33, 29, 25, 21, 17, …

    -4 -4   -4   -4   Arithmetic; Common Difference: -4

c. $\frac{1}{2}, \quad \frac{5}{8}, \quad \frac{3}{4}, \quad \frac{13}{16}, …$

   $+\frac{1}{8} + \frac{1}{8} + \frac{1}{16}$ Not Arithmetic

d. 1, 3, 5, 7, 9, 11, …

    +2 +2 +2 +2 +2   Arithmetic; Common Difference: 2

e. 1, 2, 3, 5, 8, …

    +1 +1   +2   +3   Not Arithmetic

Example 2: Find the next three terms.

a. 12, 9, 6, 3, ….   To get from number to number you subtract 3 each time.

  Your sequence would be: 12, 9, 6, 3, **0, -3, -6**

b. -2, 2, 6, 10, …   To get from number to number you add 4 each time.

  Your sequence would be: -2, 2, 6, 10, **14, 18, 22**

c. 9.5, 11.0, 12.5, 14.0   To get from number to number you add 1.5

  Your sequence would be: 9.5, 11.0, 12.5, 14.0, **15.5, 17.0, 18.5**

Practice questions!!! Book page 191 # 8-11, 13, 15

Formula for the nth Term of an Arithmetic Sequence
$$a_n = a_1 + (n - 1)d$$

a_n: n^{th} term (what you are looking for)
a_1: first term
d: common difference
n: what term you are looking for

Example 3:
A) Write an equation for the nth term of the arithmetic sequence: -12, -8, -4, 0, ...

   a_1= -12   (because it's the first term)
   d = +4   (because add 4 each time)

   $a_n = a_1 + (n - 1)d$
   $a_n = -12 + (n - 1)4$ Plug -12 in for a_1 and 4 for d
   $a_n = -12 + 4n - 4$ Distribute 4 to (n – 1)
   $\boldsymbol{a_n = 4n - 16}$ Add like terms: -12 - 4

B) Find the 6th term
   $a_n = 4n - 16$
   $a_6 = 4(6) - 16$ Remember n is what you are looking for; so plug in 6 for n
   $a_6 = 24 - 16$ Simplify: Multiply
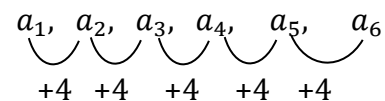 $\boldsymbol{a_6 = 8}$ Simplify: Subtract

   OR

   Since the common difference is +4, you can add 4 to the sequence until you reach the 6th term.

   -12, -8,   -4,   0,   ____ , _____

   a_1, a_2, a_3, a_4, a_5, a_6

   +4 +4   +4   +4   +4

   12, -8,   -4,   0, **4,   8**   So , $a_6 = 8$

Example 4:

A) Write an equation for the nth term of the arithmetic sequence: 3, -10, -23, -36, …

        $a_1 = 3$ (because it's the first term)

        $d = -13$ (because subtract 13 each time)

$$a_n = a_1 + (n - 1)d$$
$$a_n = 3 + (n - 1)(-13)$$ Plug 3 in for a_1 and -13 for d (notice since 13 is negative I used parentheses)
$$a_n = 3 - 13n + 13$$ Distribute -13 to (n – 1)
$$\boldsymbol{a_n = -13n + 16}$$ Add like terms: 3 + 13

B) Find the 8ᵗʰ term

$$a_n = -13n + 16$$
$$a_8 = -13(8) + 16$$ Remember n is what you are looking for; so plug in 8 for n
$$a_8 = -104 + 16$$ Simplify: Multiply
$$\boldsymbol{a_8 = -88}$$ Simplify: Add
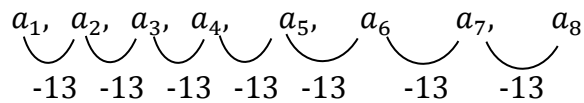
OR

Since the common difference is -13, you can subtract 13 to the sequence until you reach the 8ᵗʰ term.

3,   -10, -23, -36,   ____,   ____,   ____,   _____

$a_1, \; a_2, \; a_3, \; a_4, \quad a_5, \quad a_6 \quad a_7, \quad a_8$

-13   -13   -13   -13   -13   -13   -13

3,   -10, -23, -36,   **-49,**   **-62,**   **-75,**   **-88**   **So , $a_8 = -88$**

Practice Problems!! Book page 191 #18, 20 (Do not graph. Only write equation and find 5ᵗʰ term.) Complete this on the same sheet as your previous problems.