Algebraic Proof

Goals:
You will use algebra to write two-column proofs.
You will know the precise definition of a postulate, theorem, and counterexample.
MP 2, 3

Proof – a logical argument in which each statement you make is supported by a true statement.

Algebraic Properties of Equality	
Addition Property of Equality	$x - 2 = 5$
Subtraction Property of Equality	$x + 2 = 5$
Multiplication Property of Equality	$\frac{x}{2} = 5$
Division Property of Equality	$2x = 6$
Reflexive Property of Equality	$a = a$, $5 = 5$
Symmetric Property of Equality	If $x = 5$, then $5 = x$
Transitive Property of Equality	If $a = b$ and $b = c$, then $a = c$
Substitution Property of Equality	Replace value for an equal value
Distributive Property	$2(x - 5) = 2x - 10$

Two column proof

- Statements (steps) are organized in the left column and reasons (the reason for each step) are organized in the right column.

You start with given information and what you want to prove at the end.

Algebraic Proof Name: ______________________________

Given: $3(5x + 1) + 2 = 13x + 7$

Prove: $x = 1$

Statements	Reasons
1. $3(5x+1)+2 = 13x+7$	**Given**
2. $15x+3+2 = 13x+7$	**Distributive Property**
3. $15x+5 = 13x+7$	**Substitution Property**
4. $2x+5 = 7$	**Subtraction Property (=)**
5. $2x = 2$	**Subtraction Property (=)**
6. $x = 1$	**Division Property (=)**

Rearrange the scrambled statements and reasons and place them in the appropriate spots in the two-column proof above.

Scrambled Statements	Scrambled Reasons
1. $2x = 2$	Distributive Property
2. $x = 1$	Division Property (=)
3. $15x+3+2 = 13x+7$	Given
4. $2x+5 = 7$	Subtraction Property (=)
5. $3(5x+1)+2 = 13x+7$	Subtraction Property (=)
6. $15x+5 = 13x+7$	Substitution Property

Algebraic Proof Name: ______________________________

Given: $7 = \frac{3x+5}{2}$

Prove: $x = 3$

Statements	Reasons
1. $7 = \frac{3x+5}{2}$	**Given**
2. $2(7) = 2\left(\frac{3x+5}{2}\right)$	**Multiplication Property (=)**
3. $14 = 3x+5$	**Substitution Property**
4. $9 = 3x$	**Subtraction Property (=)**
5. $3 = x$	**Division Property (=)**
6. $x = 3$	**Symmetric Property**

Rearrange the scrambled statements and reasons and place them in the appropriate spots in the two-column proof above.

Scrambled Statements	Scrambled Reasons
1. $14 = 3x+5$	Substitution Property
2. $3 = x$	Division Property (=)
3. $2(7) = 2\left(\frac{3x+5}{2}\right)$	Given
4. $7 = \frac{3x+5}{2}$	Symmetric Property
5. $x = 3$	Multiplication Property (=)
6. $9 = 3x$	Subtraction Property (=)

Now try an algebraic proof without a bank of answers.

Given: $3x - 12 + 5 = 17$

Prove: $x = 8$

Statements	Reasons
$3x - 12 + 5 = 17$	**Given**
$3x - 7 = 17$	**Substitution**
$3x = 24$	**Addition Prop**
$x = 8$	**Division Prop**

Algebraic Proof Name: ______________________________

<u>Counterexample</u> = An example that shows a statement to be false. **(False example, can be a picture)**

Ex. Find a counter example to the following statement:
If ∠2 and ∠3 are supplementary angles, then ∠2 and ∠3 form a linear pair.

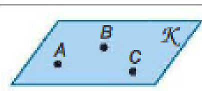
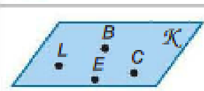
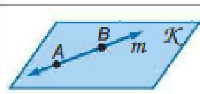
<u>Postulates:</u>	**<u>Theorems:</u>**
Accepted to be true (fundamentals of geometry)	**Proven to be true**

Examples:

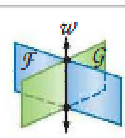
Pythagorean Theorem

Midpoint Theorem: If m is the midpoint of $\overline{AB}$**, then** $\overline{AM} \cong \overline{MB}$.

Postulates Points, Lines, and Planes

	Words	Example	
2.1	Through any two points, there is exactly one line.		Line n is the only line through points P and R.
2.2	Through any three noncollinear points, there is exactly one plane.		Plane $\mathcal{K}$ is the only plane through noncollinear points A, B, and C.
2.3	A line contains at least two points.		Line n contains points P, Q, and R.
2.4	A plane contains at least three noncollinear points.		Plane $\mathcal{K}$ contains noncollinear points L, B, C, and E.
2.5	If two points lie in a plane, then the entire line containing those points lies in that plane.		Points A and B lie in plane $\mathcal{K}$, and line m contains points A and B, so line m is in plane $\mathcal{K}$.

KeyConcept Intersections of Lines and Planes

	Words	Example	
2.6	If two lines intersect, then their intersection is exactly one point.		Lines s and t intersect at point P.
2.7	If two planes intersect, then their intersection is a line.		Planes $\mathcal{F}$ and $\mathcal{G}$ intersect in line w.

Homework:
2.6 Algebraic Proofs
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