2.7 & 2.8 – Proving Segment and Angle Relationships

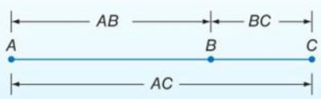
Goals:
You will prove theorems involving segment addition and congruence.

You will prove theorems involving supplementary, complementary, congruent and right angles.

Segment Addition Postulate

Words If A, B, and C are collinear, then point B is between A and C if and only if $AB + BC = AC$.

Model

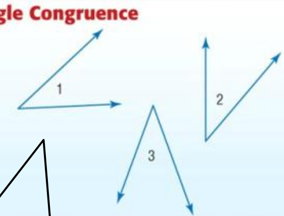


Theorem 2.5 **Properties of Angle Congruence**

Reflexive Property of Congruence
$\angle 1 \cong \angle 1$

Symmetric Property of Congruence
If $\angle 1 \cong \angle 2$, then $\angle 2 \cong \angle 1$.

Transitive Property of Congruence
If $\angle 1 \cong \angle 2$ and $\angle 2 \cong \angle 3$, then $\angle 1 \cong \angle 3$.



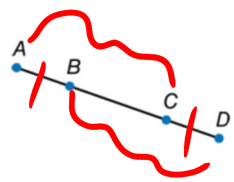
Now we can use reflexive, symmetric, and transitive with congruent segments and angles.

EXAMPLE 1 **Use the Segment Addition Postulate**

Prove that if $\overline{AB} \cong \overline{CD}$, then $\overline{AC} \cong \overline{BD}$.

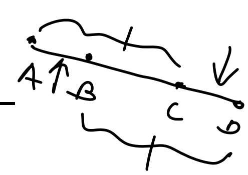
Given: $\overline{AB} \cong \overline{CD}$

Prove: $\overline{AC} \cong \overline{BD}$



Statements	Reason
$\overline{AB} \cong \overline{CD}$	given
$AB + BC = AC$	segment add post
$BC + CD = BD$	
$AB = CD$	def of ≅
$CD + BC = AC$	subst.
$AC = BD$	Subst
$\overline{AC} \cong \overline{BD}$	def. of ≅

Example 2: **Given: AC = BD** **Prove: AB = CD**



Statements	Reasons
$AC = BD$	Given
$AB + BC = AC$	Seg. add. post.
$CD + BC = BD$	
$AB + \cancel{BC} = CD + \cancel{BC}$ $-\cancel{BC} \quad -\cancel{BC}$	Subst.
$AB = CD$	subtr.

Real-World Example 2 — Proof Using Segment Congruence

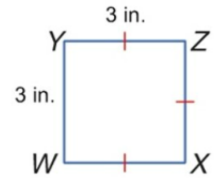
BADGE Jamie is designing a badge for her club. The length of the top edge of the badge is equal to the length of the left edge of the badge. The top edge of the badge is congruent to the right edge of the badge, and the right edge of the badge is congruent to the bottom edge of the badge. Prove that the bottom edge of the badge is congruent to the left edge of the badge.

Given: $WY = YZ$

$\overline{YZ} \cong \overline{XZ}$

$\overline{XZ} \cong \overline{WX}$

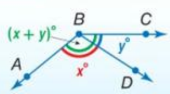
Prove: $\overline{WX} \cong \overline{WY}$



Statements	Reasons
$WY = YZ$	given
$\overline{YZ} \cong \overline{XZ}$	
$\overline{XZ} \cong \overline{WX}$	
$\overline{WY} \cong \overline{YZ}$	def of $\cong$
$\overline{WY} \cong \overline{XZ}$	subst.
$\overline{WY} \cong \overline{WX}$	subst.
$\overline{WX} \cong \overline{WY}$	symm. prop.

Postulate 2.11 Angle Addition Postulate

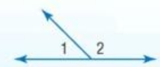
D is in the interior of $\angle ABC$ if and only if $m\angle ABD + m\angle DBC = m\angle ABC$.



Theorems

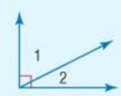
2.3 Supplement Theorem If two angles form a linear pair, then they are supplementary angles.

Example $m\angle 1 + m\angle 2 = 180$



2.4 Complement Theorem If the noncommon sides of two adjacent angles form a right angle, then the angles are complementary angles.

Example $m\angle 1 + m\angle 2 = 90$

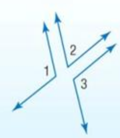


Theorems

2.6 Congruent Supplements Theorem
Angles supplementary to the same angle or to congruent angles are congruent.

Abbreviation $\angle$s suppl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

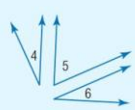
Example If $m\angle 1 + m\angle 2 = 180$ and $m\angle 2 + m\angle 3 = 180$, then $\angle 1 \cong \angle 3$.



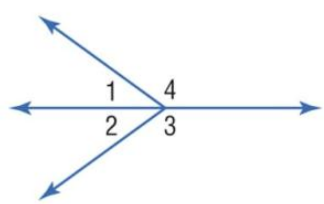
2.7 Congruent Complements Theorem
Angles complementary to the same angle or to congruent angles are congruent.

Abbreviation $\angle$s compl. to same $\angle$ or $\cong$ $\angle$s are $\cong$.

Example: If $m\angle 4 + m\angle 5 = 90$ and $m\angle 5 + m\angle 6 = 90$, then $\angle 4 \cong \angle 6$.



In the figure, $\angle 1$ and $\angle 4$ form a linear pair, and $m\angle 3 + m\angle 1 = 180$. Prove that $\angle 3$ and $\angle 4$ are congruent.



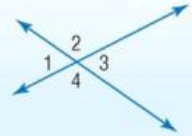
Statements	Reasons
$\angle 1$ & $\angle 4$ form a linear pair	given
$m\angle 3 + m\angle 1 = 180$	
$m\angle 1 + m\angle 4 = 180$	Suppl. Thm
$m\angle 1 + m\angle 4 = m\angle 3 + m\angle 1$	subst.
$-m\angle 1 \qquad -m\angle 1$	
$m\angle 4 = m\angle 3$	subtr.
$m\angle 3 = m\angle 4$	symm.
$\angle 3 \cong \angle 4$	def. of $\cong$

Theorem 2.8 **Vertical Angles Theorem**

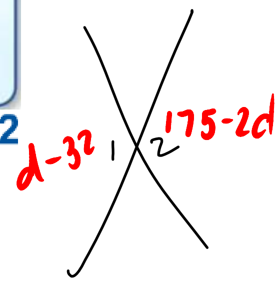
If two angles are vertical angles, then they are congruent.

Abbreviation Vert. ∠s are ≅.

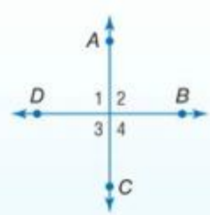
Example $\angle 1 \cong \angle 3$ and $\angle 2 \cong \angle 4$



If $\angle 1$ and $\angle 2$ are vertical angles and $m\angle 1 = d - 32$ and $m\angle 2 = 175 - 2d$, find $m\angle 1$ and $m\angle 2$. Justify each step.



Statements	Reasons
$\angle 1$ & $\angle 2$ are vertical ∠'s $m\angle 1 = d-32$ $m\angle 2 = 175-2d$	given
$\angle 1 \cong \angle 2$	Vert. ∠'s thm
$m\angle 1 = m\angle 2$	def of ≅
$d-32 = 175-2d$	subst.
$+2d \quad +2d$ $3d-32=175$	add prop.
$+32 \quad +32$ $\frac{3d}{3} = 207$	add prop
$d = 69$	div. prop
$m\angle 1 = 69-32$, $m\angle 2 = 175-2\cdot 69$ $m\angle 1 = 37$, $m\angle 2 = 37$	subst.

Theorem	Example
2.9 Perpendicular lines intersect to form four right angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ∠s.	
2.10 All right angles are congruent. **Example** If $\angle 1$, $\angle 2$, $\angle 3$, and $\angle 4$ are rt. ∠s, then $\angle 1 \cong \angle 2 \cong \angle 3 \cong \angle 4$.	
2.11 Perpendicular lines form congruent adjacent angles. **Example** If $\overleftrightarrow{AC} \perp \overleftrightarrow{DB}$, then $\angle 1 \cong \angle 2$, $\angle 2 \cong \angle 4$, $\angle 3 \cong \angle 4$, and $\angle 1 \cong \angle 3$.	
2.12 If two angles are congruent and supplementary, then each angle is a right angle. **Example** If $\angle 5 \cong \angle 6$ and $\angle 5$ is suppl. to $\angle 6$, then $\angle 5$ and $\angle 6$ are rt. ∠s.	
2.13 If two congruent angles form a linear pair, then they are right angles. **Example** If $\angle 7$ and $\angle 8$ form a linear pair, then $\angle 7$ and $\angle 8$ are rt. ∠s.